HOMEWORK 10 – ANSWERS TO (MOST) PROBLEMS

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Section 5.1: Areas and Distances

5.1.2.

(a) (i) $\Delta x = 2$, so

$$L_6 = f(0)(2)+f(2)(2)+f(4)(2)+f(6)(2)+f(8)(2)+f(10)(2) = 18+\frac{52}{3}+\frac{50}{3}+\frac{44}{3}+12+8 = \frac{260}{3} \approx 86.67$$

(ii)

$$R_6 = f(2)(2)+f(4)(2)+f(6)(2)+f(8)(2)+f(10)(2)+f(12)(2) = \frac{52}{3}+\frac{50}{3}+\frac{44}{3}+12+8+2 = \frac{212}{3} \approx 70.67$$

(iii)

$$M_6 = f(1)(2)+f(3)(2)+f(5)(2)+f(7)(2)+f(9)(2)+f(11)(2) = 18+17+15+13+10+\frac{16}{3} = \frac{235}{3} \approx 78.33$$

(b) Overestimate
(c) Underestimate
(d) M_6 (just right, does not overshoot, like L_6, but not undershoot either, like R_6)

5.1.5.

(a) If $n = 3$, then $\Delta x = 1$, and if $n = 6$, $\Delta x = \frac{1}{2}$, so:

$$R_3 = f(0)(1) + f(1)(1) + f(2)(1) = 1 + 2 + 5 = 8$$

$$\begin{aligned} R_6 =& f(-0.5)(0.5) + f(0)(0.5) + f(0.5)(0.5) + f(1)(0.5) + f(1.5)(0.5) + f(2)(0.5) \\ =& 1.25(0.5) + 1(0.5) + 1.25(0.5) + 2(0.5) + 3.25(0.5) + 5(0.5) \\ =& 6.875 \end{aligned}$$

(b)

$$L_3 = f(-1)(1) + f(0)(1) + f(1)(1) = 2 + 1 + 2 = 5$$

$$\begin{aligned} L_6 =& f(-1)(0.5) + f(-0.5)(0.5) + f(0)(0.5) + f(0.5)(0.5) + f(1)(0.5) + f(1.5)(0.5) \\ =& 2(0.5) + 1.25(0.5) + 1(0.5) + 1.25(0.5) + 2(0.5) + 3.25(0.5) \\ =& 5.375 \end{aligned}$$

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(c)
$$M_3 = f(-0.5)(1) + f(0.5)(1) + f(1.5)(1) = 5.75$$

$$M_6 = f(-0.75)(0.5)+f(-0.25)(0.5)+f(0.25)(0.5)+f(0.75)(0.5)+f(1.25)(0.5)+f(1.75)(0.5) = 5.9375$$

(d) M_6

5.1.13. Here $n = 6$ and $\Delta x = 0.5$

$$\begin{aligned} L_6 =& v(0)(0.5) + v(0.5)(0.5) + v(1)(0.5) + v(1.5)(0.5) + v(2)(0.5) + v(2.5)(0.5) \\ =& 0(0.5) + 6.2(0.5) + 10.8(0.5) + 14.9(0.5) + 18.1(0.5) + 19.4(0.5) \\ =& 34.7 \end{aligned}$$

$$\begin{aligned} R_6 =& v(0.5)(0.5) + v(1)(0.5) + v(1.5)(0.5) + v(2)(0.5) + v(2.5)(0.5) + v(3)(0.5) \\ =& 6.2(0.5) + 10.8(0.5) + 14.9(0.5) + 18.1(0.5) + 19.4(0.5) + 20.2(0.5) \\ =& 44.8 \end{aligned}$$

5.1.17. The midpoint sum seems to best approximate the area:

$$M_6 = v(0.5)(1)+v(1.5)(1)+v(2.5)(1)+v(3.5)(1)+v(4.5)(1)+v(5.5)(1) = 50+40+30+18+10+5 = 153ft$$

5.1.19. Here $\Delta x = \frac{2}{n}$ and $x_i = 1 + \frac{2i}{n}$, so:

$$\lim_{n\to\infty} \sum_{i=1}^{\infty} \frac{2}{n} \frac{2\left(1 + \frac{2i}{n}\right)}{\left(1 + \frac{2i}{n}\right)^2 + 1}$$

5.1.20. Here $\Delta x = \frac{\pi}{n}$ and $x_i = \frac{\pi i}{n}$, so:

$$\lim_{n\to\infty} \sum_{i=1}^{\infty} \frac{\pi}{n} \sqrt{\sin\left(\frac{\pi i}{n}\right)}$$

5.1.22. The area under the curve of $f(x) = x^{10}$ from 5 to 7 (or, if you want, the area under the curve of $f(x) = (x+5)^{10}$ from 0 to 2)

5.1.23. The area under the curve of $f(x) = \tan(x)$ from 0 to $\frac{\pi}{4}$

Section 5.2: The definite integral

5.2.18. $\int_{\pi}^{2\pi} \frac{\cos(x)}{x} dx$

5.2.21. First of all, $a = 2$, $b = 5$, $\Delta x = \frac{3}{n}$ and $x_i = 2 + \frac{3i}{n}$

$$\begin{aligned}
\int_2^5 4 - 2x dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(4 - 2\left(2 + \frac{3i}{n}\right)\right) \\
&= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(4 - 4 - \frac{6i}{n}\right) \\
&= \lim_{n\to\infty} \frac{3}{n} \left(\sum_{i=1}^{n} \frac{-6}{n} i\right) \\
&= \lim_{n\to\infty} \frac{-18}{n^2} \left(\sum_{i=1}^{n} i\right) \\
&= \lim_{n\to\infty} \frac{-18}{n^2} \left(\frac{n(n+1)}{2}\right) \\
&= \lim_{n\to\infty} \frac{-9n^2}{n^2 + n} \\
&= \lim_{n\to\infty} \frac{-9n^2}{n^2\left(1 + \frac{1}{n}\right)} \\
&= \lim_{n\to\infty} \frac{-9}{\left(1 + \frac{1}{n}\right)} \\
&= -9
\end{aligned}$$

5.2.23. First of all, $a = -2$, $b = 0$, $\Delta x = \frac{2}{n}$ and $x_i = -2 + \frac{2i}{n}$

$$
\begin{aligned}
\int_{-2}^{0} x^2 + x dx =& \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(-2+\frac{2i}{n}\right)^2 + \left(-2+\frac{2i}{n}\right) \\
=& \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left(4-\frac{8i}{n}+\frac{4i^2}{n^2}-2+\frac{2i}{n}\right) \\
=& \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left(\frac{4i^2}{n^2}-\frac{6i}{n}+2\right) \\
=& \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left(\frac{4i^2}{n^2}\right)-\frac{2}{n}\sum_{i=1}^{n}\left(\frac{6i}{n}\right)+\frac{2}{n}\sum_{i=1}^{n}(2) \\
=& \lim_{n\to\infty} \frac{8}{n^3}\left(\sum_{i=1}^{n} i^2\right)-\frac{12}{n^2}\left(\sum_{i=1}^{n} i\right)+\frac{4}{n}\left(\sum_{i=1}^{n} 1\right) \\
=& \lim_{n\to\infty} \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)-\frac{12}{n^2}\left(\frac{n(n+1)}{2}\right)+\frac{4}{n}n \\
=& \lim_{n\to\infty} \frac{8}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)-6\left(1+\frac{1}{n}\right)+4 \\
=& \frac{4}{3}\times 2-6+4 \\
=& \frac{8}{3}-2 \\
=& \frac{2}{3}
\end{aligned}
$$

5.2.34.

(a) 4 (the area of the large triangle)

(b) -2π (minus the area of the semicircle)

(c) $4-2\pi+\frac{1}{2}=\frac{9}{2}-2\pi$ (the area of the large triangle minus the area of the semicircle plus the area of the small triangle)

5.2.37. $3+\frac{9\pi}{4}$ (the area of a rectangle plus the area of a quarter of a circle of radius 3)

5.2.47.

$$\int_{-2}^{2} f(x)dx+\int_{2}^{5} f(x)dx-\int_{-2}^{-1} f(x)dx=\int_{-2}^{5} f(x)dx-\int_{-2}^{-1} f(x)dx=\int_{-2}^{5} f(x)dx+\int_{-1}^{-2} f(x)dx=\int_{-1}^{5} f(x)dx$$

5.2.54. $m \leq f(x) \leq M$, so integrating from 0 to 2, we get $2m \leq \int_0^2 f(x)dx \leq 2M$

5.2.56. On $[0,1]$, $x^2 \leq x$, so $1+x^2 \leq 1+x$, so $\sqrt{1+x^2} \leq \sqrt{1+x}$, so integrating from 0 to 1, we get $\int_0^1 \sqrt{1+x^2}dx \leq \int_0^1 \sqrt{1+x}dx$

Section 5.3: The Fundamental Theorem of Calculus

5.3.7. $\frac{1}{x^3+1}$

5.3.15. $\sec^2(x)\sqrt{\tan(x)+\sqrt{\tan(x)}}$

5.3.17. $3\frac{(1-3x)^3}{1+(1-3x)^2}$

5.3.27. $-\frac{37}{6}$ (Write $(u+2)(u-3) = u^2 - u - 6$)

5.3.35. $\ln(2) + 7$ (Write $\frac{v^3+3v^6}{v^4} = \frac{1}{v} + \frac{3}{v^2}$, whose antiderivative is $\ln|v| - \frac{3}{v}$)

5.3.37. $\frac{1}{e+1} + (e-1)$ (Antiderivative is $\frac{x^{e+1}}{e+1} + e^x$)

5.3.39. $8\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{4\pi}{3}$ (antiderivative is $8\tan^{-1}(t)$)